Board Level Exercise

Type (I) : Very Short Answer Type Questions : **[01 Mark Each]**

1. Find the first five terms of the sequence for which $t_1 = 1$, $t_2 = 2$ and $t_{n+2} = t_n + t_{n+1}$

2. How many terms are there in the A.P. 20, 25, 30 100

3. Is 55 a term of the sequence 1, 3, 5, 7, ... ? If yes find which term it is.

4. Find the sum of the series 99 + 95 + 91 + 87 + ... to 20 terms

5. If sum to n terms of sequence be $n^2 + 2n$, then find first term, common difference and also find sequence

6. A boy agrees to work at the rate of one rupee the first day, two rupees the second day, four rupees the third day, eight rupees the fourth day and so on in the month of April. How much would he get on the 20th of April.

7. How many terms are there in the G.P.
5, 20, 80, ... 5120 ?

8. The seventh term of a G.P. is 8 times the fourth term. Find the G.P. when its 5th term is 48

9. How many terms of the series
$1 + 3 + 3^2 + 3^3 + ...$ must be taken to make 3280 ?

10. Find $1 + \frac{x}{(1+x)} + \left(\frac{x}{1+x}\right)^2 + ...$ to ∞ if $x > 0$

11. Find $4^3 + 5^3 + 6^3 + ... 10^3$

12. If $0 < \theta < \frac{\pi}{2}$, then prove that $\tan\theta + \cot\theta \geq 2$

Type (II) : Short Answer Type Questions : **[02 Marks Each]**

13. Find the A.P. whose 7th and 13th terms are respectively 34 and 64.

14. Find the sum of all even number between 101 to 999.

15. Insert 4 G.M's between 5 and 160.

16. Insert 7 A.M.'s between 2 and 34

17. If m times the m th term of an A.P. is equal to n times the nth term, find its (m + n)th term.

18. Three numbers are in A.P. their sum is 27 and sum of their squares is 275. Find the numbers.

19. If a, b, c are in A.P., prove that b + c, c + a, a + b are also in A.P.

20. If a, b, c are in A.P., show that
(i) $2(a - b) = (a - c) = 2(b - c)$ (ii) $(a - c)^2 = 4(b^2 - ac)$

21. Find the sum of the series $(a + b)^2 + (a^2 + b^2) + (a - b)^2 + ... +$ to n terms.

22. Find the least value of n such that 1 + 3 + 5 + 7 + ... to n terms $\geq$ 500

23. Find the sum to n terms of the series 8 + 88 + 888 + ...

24. Express $0.\overline{54}$ as a rational number

25. Prove that in an infinite G.P. whose common ratio r is numerically less than one, the ratio of any terms to the sum of all the succeeding terms is $\frac{1-r}{r}$

26. Find $\sum_{r=1}^{n}(3^r - 2^r)$

27. $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}$ + ... to n terms.

Type (III) : Long Answer Type Questions: **[04 Mark Each]**

28. If there are (2n + 1) terms in an A.P., then prove that the ratio of the sum of odd terms and thesum of even terms is n + 1 : n

29. If a, b, c are in A.P. and x, y, z are in G.P. show that $x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1$

30. If the A.M. and G.M. between two number be 5 and 3 respectively, find the numbers.

31. Prove that the number 1111......1 (91 digits) is not a prime number.

32. Find the sum of the n term of the series whose nth term is $12n^2 - 6n + 5$

33. The continued product of three numbers in G.P. is 216 and the sum of the product of them in pairs is 156; Find the number

34. If $(a)^{1/x} = (b)^{1/y} = (c)^{1/z}$ and a, b, c are in G.P., then prove that x, y, z are in A.P.

35. If a, b, c, d four distinct positive quantities in G.P. then show that a + d > b + c

36. If the sum of first 10 terms of an A.P. is 140 and the sum of first 16 terms is 120, find the sum of n terms

37. The first and last term of an A.P. are a and ℓ respectively. If S be the sum of all the term of the A.P., show that the common difference is $\frac{\ell^2 - a^2}{2S - (\ell + a)}$

38. The sum of four integers in A.P. is 24 and their product is 945. Find the numbers.

39. In a set of four numbers, the first three are in G.P. and the last three are in A.P. with a common difference of 6. If the first number is same as the fourth find the four number.

Type (IV) : Very Long Answer Type Questions: **[06 Mark Each]**

40. If S_1, S_2, S_3, ... S_P be the sum of n terms of Arithmetic progressions whose first terms are respectively. 1, 2, 3 ...... and common difference are 1, 2, 3 .....prove that $S_1 + S_2 + S_3 + ... S_p = \frac{np}{4}(n + 1)(p + 1)$

41. A number consists of three digits in G.P. the sum of the right hand and left hand digit exceeds twice the middle digit by 1 and the sum of the left hand and middle digits is two third of the sum of the middle and right hand digits. Find the number

42. Find 30th term of the series 3 + 5 + 9 + 15 + 23 ...... ?

43. If a be one A.M. and g_1 and g_2 be two geometric means between b and c, prove that $g_1^3 + g_2^3$ = 2abc

44. If x + y + z = 1 and x, y, z are positive numbers show that (1 – x) (1 – y) (1 – z) ≥ 8xyz

45. If a + b + c = 3 and a > 0, b > 0, c > 0, find the greatest value of $a^2b^3c^2$.

46. Four different integers form an increasing A.P. one of these number is equal to the sum of the squares of the other three numbers find the numbers.

47. Find the sum of n terms of the series 1 + 5 + 11 + 19 + 29 + ...

48. How many terms are identical in the two arithmetic progressions 2, 4, 6, 8,..... Up to 100 term and 3, 6, 9, ... up to 80 terms.

49. The p th term of an A.P. is a and qth term is b, prove that sum of its (p + q) terms is $\frac{p+q}{2}\left(a+b+\frac{a-b}{p+q}\right)$

50. A man arranges to pay off a debt of Rs. 3600 by 40 annual installment which are in A.P., when 30 of the installments are paid he dies leaving one third of the debt unpaid find the value of the 8th installment.

51. Prove that the sum of latter half of 2n terms of a series in A.P. is equal to one third of the sum of the first 3n terms.

Exercise # 1

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Arithmetic Progression

A-1. In an A.P. the third term is four times the first term, and the sixth term is 17 ; find the series.

A-2. The third term of an A.P. is 18, and the seventh term is 30 ; find the sum of 17 terms.

A-3. How many terms of the series – 9, – 6, – 3, ... must be taken that the sum may be 66 ?

A-4. Find the number of integers between 100 & 1000 that are
(i) divisible by 7 (ii) not divisible by 7

A-5. Find the sum of all those integers between 100 and 800 each of which on division by 16 leaves the remainder 7.

A-6. Find the sum of 35 terms of the series whose p^{th} term is $\frac{p}{7}$ + 2.

A-7. The sum of three numbers in A.P. is 27, and their product is 504, find them.

A-8. If a, b, c are in A.P., then show that:
(i) a^2 (b + c), b^2 (c + a), c^2 (a + b) are also in A.P.
(ii) b + c – a, c + a – b, a + b – c are in A.P.

A-9. The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer.

Section (B) : Geometric Progression

B-1. The third term of a G.P. is the square of the first term. If the second term is 8, find its sixth term.

B-2. The continued product of three numbers in G.P. is 216, and the sum of the products of them in pairs is 156; find the numbers

B-3. If the p^{th} , q^{th} , r^{th} terms of a G.P. be a, b, c respectively, prove that $a^{q-r}\, b^{r-p}\, c^{p-q} = 1$.

B-4. The sum of infinite number of terms of a G.P. is 4 and the sum of their cubes is 192. Find the series.

B-5. If a, b, c, d are in G.P., prove that :

(i) $(a^2 - b^2), (b^2 - c^2), (c^2 - d^2)$ are in G.P.

(ii) $\frac{1}{a^2 + b^2}, \frac{1}{b^2 + c^2}, \frac{1}{c^2 + d^2}$ are in G.P.

Section (C) : Harmonic and Arithmetic Geometric Progression

C-1. Find the 4^{th} term of an H.P. whose 7^{th} term is $\frac{1}{20}$ and 13^{th} term is $\frac{1}{38}$.

C-2. Sum the following series

(i) $1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots\ldots..$ to n terms.

(ii) $1 + \frac{3}{4} + \frac{7}{16} + \frac{15}{64} + \frac{31}{256} + \ldots..$ to infinity.

C-3. Find the sum of n terms of the series the r^{th} term of which is $(2r + 1)2^r$.

Section (D) : Means, Inequalities A.M. ≥ G.M. ≥ H.M

D-1. The arithmetic mean of two numbers is 6 and their geometric mean G and harmonic mean H satisfy the relation $G^2 + 3H = 48$. Find the two numbers.

D-2. If between any two quantities there be inserted two arithmetic means A_1, A_2; two geometric means G_1, G_2; and two harmonic means H_1, H_2 then prove that $G_1 G_2 : H_1 H_2 = A_1 + A_2 : H_1 + H_2$.

D-3. If x > 0, then find greatest value of the expression $\frac{x^{100}}{1 + x + x^2 + x^3 + \ldots.. + x^{200}}$.

D-4. Using the relation A.M. ≥ G.M. prove that

(i) $\tan\theta + \cot\theta \geq 2$; if $0 < \theta < \frac{\pi}{2}$

(ii) $(x^2y + y^2z + z^2x)\,(xy^2 + yz^2 + zx^2) \geq 9x^2y^2z^2$. (x, y, z are positive real number)

(iii) (a + b) . (b + c) . (c + a) > abc ; if a, b, c are positive real numbers

Section (E) : Method of difference, $t_n = v_n - v_{n-1}$

E-1. Find the sum to n-terms of the sequence.
(i) 1 + 5 + 13 + 29 + 61 + ... up to n terms
(ii) 3 + 33 + 333 + 3333 + ... up to n terms

E-2. Find the sum to n-terms of the sequence.

(i) $\frac{1}{1.3.5} + \frac{1}{3.5.7} + \frac{1}{5.7.9} + ...$ (ii) $1 . 3 . 2^2 + 2 . 4 . 3^2 + 3 . 5 . 4^2 + ...$

Section (F) : Miscellaneous

F-1. The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1 & the third is increased by 1, we obtain three consecutive terms of a G.P., find the numbers.

F-2. If the p^{th}, q^{th} & r^{th} terms of an AP are in GP. Find the common ratio of the GP.

F-3. Find the sum of the n terms of the series whose nth term is
(i) $n(n + 2)$ (ii) $3^n - 2^n$

PART - II : OBJECTIVE QUESTIONS

*** Marked Questions may have more than one correct option.**

Section (A) : Arithmetic Progression

A-1. The first term of an A.P. of consecutive integer is $p^2 + 1$. The sum of $(2p + 1)$ terms of this series can be expressed as
(A) $(p + 1)^2$ (B) $(2p + 1)(p + 1)^2$ (C) $(p + 1)^3$ (D) $p^3 + (p + 1)^3$

A-2. If $a_1, a_2, a_3,........$ are in A.P. such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then $a_1 + a_2 + a_3 + ... + a_{23} + a_{24}$ is equal to
(A) 909 (B) 75 (C) 750 (D) 900

A-3. If the sum of the first 2n terms of the A.P. 2, 5, 8, ..., is equal to the sum of the first n terms of the A.P. 57, 59, 61, ..., then n equals
(A) 10 (B) 12 (C) 11 (D) 13

A-4. The sum of integers from 1 to 100 that are divisible by 2 or 5 is
(A) 2550 (B) 1050 (C) 3050 (D) none of these

A-5. The interior angles of a polygon are in A.P. If the smallest angle is 120º & the common difference is 5º, then the number of sides in the polygon is:
(A) 7 (B) 9 (C) 16 (D) none of these

A-6. Consider an A.P. with first term 'a' and the common difference 'd'. Let S_k denote the sum of its first K terms. If $\frac{S_{kx}}{S_x}$ is independent of x, then
(A) a = d/2 (B) a = d (C) a = 2d (D) none of these

A-7. If $x \in R$, the numbers $5^{1+x} + 5^{1-x}$, a/2, $25^x + 25^{-x}$ form an A.P. then 'a' must lie in the interval:
(A) [1, 5] (B) [2, 5] (C) [5, 12] (D) $[12, \infty)$

A-8. There are n A.M's between 3 and 54, such that the 8th mean: $(n - 2)^{th}$ mean:: 3: 5. The value of n is.
(A) 12 (B) 16 (C) 18 (D) 20

A-9. The sum of the series $\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + ... + \frac{1}{\log_{2^n} 4}$ is

(A) $\frac{1}{2}n(n + 1)$ (B) $\frac{1}{12}n(n + 1)(2n + 1)$

(C) $\frac{1}{n(n+1)}$ (D) $\frac{1}{4}n(n + 1)$

A-10*. If $a_1, a_2, ..., a_n$ are distinct terms of an A.P., then
(A) $a_1 + 2a_2 + a_3 = 0$ (B) $a_1 - 2a_2 + a_3 = 0$
(C) $a_1 + 3a_2 - 3a_3 - a_4 = 0$ (D) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

A-11*. If x, |x + 1|, |x – 1| are three terms of an A.P., then its sum upto 20 terms is –
(A) 180 (B) 350 (C) 90 (D) 720

Section (B) : Geometric Progression

B-1. The third term of a G.P is 4. The product of the first five terms is
(A) 4^3 (B) 4^5 (C) 4^4 (D) none of these

B-2. If S is the sum to infinity of a G.P. whose first term is 'a', then the sum of the first n terms is
(A) $S\left(1-\frac{a}{S}\right)^n$ (B) $S\left[1-\left(1-\frac{a}{S}\right)^n\right]$ (C) $a\left[1-\left(1-\frac{a}{S}\right)^n\right]$ (D) none of these

B-3. Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is 3/4, then:
(A) $a = \frac{7}{4}, r = \frac{3}{7}$ (B) $a = 2, r = \frac{3}{8}$ (C) $a = \frac{3}{2}, r = \frac{1}{2}$ (D) $a = 3, r = \frac{1}{4}$

B-4. For a sequence $\{a_n\}$, $a_1 = 2$ and $\frac{a_{n+1}}{a_n} = \frac{1}{3}$. Then $\sum_{r=1}^{20} a_r$ is
(A) $\frac{20}{2}$ [4 + 19 × 3] (B) $3\left(1-\frac{1}{3^{20}}\right)$ (C) $2(1 - 3^{20})$ (D) none of these

B-5. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. if a < b < c and a + b + c = $\frac{3}{2}$, then the value of a is
(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

B-6. α, β be the roots of the equation $x^2 - 3x + a = 0$ and γ, δ the roots of $x^2 - 12x + b = 0$ and numbers $\alpha, \beta, \gamma, \delta$ (in this order) form an increasing G.P., then
(A) a = 3, b = 12 (B) a = 12, b = 3 (C) a = 2, b = 32 (D) a = 4, b = 16

B-7. The rational number, which equals the number $2.\overline{357}$ with recurring decimal is
(A) $\frac{2355}{1001}$ (B) $\frac{2379}{997}$ (C) $\frac{2355}{999}$ (D) none of these

B-8*. If sum of the infinite G.P., p, 1, $\frac{1}{p}, \frac{1}{p^2}, \frac{1}{p^3}$,....... is $\frac{9}{2}$, then value of p is
(A) 3 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{1}{3}$

B-9*. Indicate the correct alternative(s), for $0 < \phi < \pi/2$, if:
$x = \sum_{n=0}^{\infty} \cos^{2n}\phi$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$, $z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$, then:
(A) xyz = xz + y (B) xyz = xy + z (C) xyz = x + y + z (D) xyz = yz + x

Section (C) : Harmonic and Arithmetic Geometric Progression

C-1. Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are:
(A) not in A.P./G.P./H.P. (B) in A.P.
(C) in G.P. (D) in H.P.

C-2. If the sum of the roots of the quadratic equation, $ax^2 + bx + c = 0$ is equal to sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in
(A) A.P. (B) G.P. (C) H.P. (D) none of these

C-3. If $3 + \frac{1}{4}(3 + d) + \frac{1}{4^2}(3 + 2d) + ... +$ upto $\infty = 8$, then the value of d is:
(A) 9 (B) 5 (C) 1 (D) none of these

Section (D) : Means, Inequalities A.M. ≥ G.M. ≥ H.M

D-1. If A, G & H are respectively the A.M., G.M. & H.M. of three positive numbers a, b, & c, then the equation whose roots are a, b, & c is given by:
(A) $x^3 - 3Ax^2 + 3G^3x - G^3 = 0$ (B) $x^3 - 3Ax^2 + 3(G^3/H)x - G^3 = 0$
(C) $x^3 + 3Ax^2 + 3(G^3/H)x - G^3 = 0$ (D) $x^3 - 3Ax^2 - 3(G^3/H)x + G^3 = 0$

D-2*. If the arithmetic mean of two positive numbers a & b (a > b) is twice their geometric mean, then a: b is:
(A) $2 + \sqrt{3} : 2 - \sqrt{3}$ (B) $7 + 4\sqrt{3} : 1$ (C) $1 : 7 - 4\sqrt{3}$ (D) $2 : \sqrt{3}$

D-3. If a, b, c, d are positive real numbers such that a + b + c + d = 2, then M = (a + b) (c + d) satisfies the relation:
(A) $0 \le M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$

D-4. If $a_1, a_2, a_3, ..., a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + a_3 + ... + a_{n-1} + 2a_n$ is
(A) $n(2c)^{1/n}$ (B) $(n + 1)c^{1/n}$ (C) $2nc^{1/n}$ (D) $(n + 1)(2c)^{1/n}$

Section (E) : Method of difference, $t_n = v_n - v_{n-1}$

E-1*. If $\sum_{r=1}^{n} r(r+1)(2r + 3) = an^4 + bn^3 + cn^2 + dn + e$, then
(A) a + c = b + d (B) e = 0
(C) a, b – 2/3, c – 1 are in A.P. (D) c/a is an integer

Section (F) : Miscellaneous

F-1. Suppose a, b, c are in A.P. & $|a|, |b|, |c| < 1$. If $x = 1 + a + a^2 + ...$ to ∞ ; $y = 1 + b + b^2 + ...$ to ∞ & $z = 1 + c + c^2 + ...$ to ∞, then x, y, z are in:
(A) A.P. (B) G.P. (C) H.P. (D) none

F-2. The sum of the first n-terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n(n+1)^2}{2}$, when n is even. When n is odd, the sum is
(A) $\frac{n(n+1)^2}{4}$ (B) $\frac{n^2(n+2)}{4}$ (C) $\frac{n^2(n+1)}{2}$ (D) $\frac{n(n+2)^2}{4}$

F-3. If $a^x = b^y = c^z = d^t$ and a, b, c, d are in G.P., then x, y, z, t are in
(A) A.P. (B) G.P. (C) H.P. (D) none of these

F-4. The sum $\sum_{r=2}^{\infty} \frac{1}{r^2 - 1}$ is equal to:
(A) 1 (B) 3/4 (C) 4/3 (D) none of these

PART - III : ASSERTION / REASONING

1. **STATEMENT-1 :** The series for which sum to n terms, S_n, is given by $S_n = 5n^2 + 6n$ is an A.P.
STATEMENT-2 : The sum to n terms of an A.P. having non–zero common difference is a quadratic in n, i.e., $an^2 + bn$.
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

2. **STATEMENT-1 :** 1, 2, 4, 8, ... is a G.P., 4, 8, 16, 32 is a G.P. and 1 + 4, 2 + 8, 4 + 16, 8 + 32, ... is also a G.P.
STATEMENT-2 : Let general term of a G.P. (with positive terms) with common ratio r be T_{k+1} and general term of another G.P. (with positive terms) with common ratio r be T'_{k+1}, then the series whose general term $T''_{k+1} = T_{k+1} + T'_{k+1}$ is also a G.P. with common ratio r.
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

3. **STATEMENT-1 :** The sum of the first 30 terms of the sequence 1,2,4,7,11,16, 22,...... is 4520.
STATEMENT-2 : If the successive differences of the terms of a sequence form an A.P., then general term of sequence is of the form $an^2 + bn + c$.
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

4. **STATEMENT-1 :** 3,6,12 are in G.P., then 9,12,18 are in H.P.
STATEMENT-2 : If three consecutive terms of a G.P. are positive and if middle term is added in these terms, then resultant will be in H.P.
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

5. **STATEMENT-1 :** Minimum value of $\dfrac{\sin^3 x + \cos^3 x + 3\sin^2 x + 3\sin x + 2}{(\sin x + 1)\cos x}$ for $x \in \left[0, \dfrac{\pi}{2}\right)$ is 3
STATEMENT-2 : The least value of $a \sin\theta + b \cos\theta$ is $-\sqrt{a^2 + b^2}$
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

Exercise # 2

PART - I : SUBJECTIVE QUESTIONS

1. Find the sum in the n^{th} group of sequence,
(i) (1), (2, 3); (4, 5, 6, 7); (8, 9,........, 15); ....
(ii) (1), (2, 3, 4), (5, 6, 7, 8, 9),........

2. Show that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ cannot be the terms of a single A.P.

3. If the sum of the first m terms of an A.P. is equal to the sum of either the next n terms or the next p terms, then prove that $(m + n)\left(\frac{1}{m}-\frac{1}{p}\right) = (m + p)\left(\frac{1}{m}-\frac{1}{n}\right)$.

4. Find the sum of the series $\frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} + \ldots$ up to ∞

5. If $0 < x < \pi$ and the expression
$\exp\{(1 + |\cos x| + \cos^2 x + |\cos^3 x| + \cos^4 x + \ldots\ \text{upto } \infty) \log_e 4\}$
satisfies the quadratic equation $y^2 - 20y + 64 = 0$, then find the value of x.

6. In a circle of radius R a square is inscribed, then a circle is inscribed in the square, a new square in the circle and so on for n times. Find the limit of the sum of areas of all the circles and the limit of the sum of areas of all the squares as $n \to \infty$.

7. Given that α, γ are roots of the equation $Ax^2 - 4x + 1 = 0$ and β, δ the roots of the equation $Bx^2 - 6x + 1 = 0$, find values of A and B, such that α, β, γ & δ are in H.P.

8. (i) If $\frac{a+be^y}{a-be^y} = \frac{b+ce^y}{b-ce^y} = \frac{c+de^y}{c-de^y}$, then show that a,b,c,d are in G.P.

(ii) If $\frac{1}{\sqrt{b}+\sqrt{c}}$, $\frac{1}{\sqrt{c}+\sqrt{a}}$, $\frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P., then show that 9^{ax+1}, 9^{bx+1}, 9^{cx+1}, $x \neq 0$ are in G.P.

9. If a, b, c are positive real numbers, then prove that $b^2c^2 + c^2a^2 + a^2b^2 \geq abc(a + b + c)$.

10. If a, b, c are positive real numbers and sides of the triangle, then prove that
$(a + b + c)^3 \geq 27 (a + b - c)(c + a - b)(b + c - a)$

11. Sum the following series to n terms.
(i) $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$ (ii) $\frac{n}{1.2.3} + \frac{n-1}{2.3.4} + \ldots\ldots + \frac{1}{n(n+1)(n+2)}$.

12. Sum of the following series
(i) $1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + \ldots\ldots\ \infty$.

(ii) $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots\ldots\infty$

13. The sum of the first ten terms of an AP is 155 & the sum of first two terms of a GP is 9. The first term of the AP is equal to the common ratio of the GP & the first term of the GP is equal to the common difference of the AP. Find the two progressions.

14. Let $a_1, a_2, \ldots\ldots$, be positive real numbers in geometric progression. For each n, let A_n, G_n, H_n be respectively the arithmetic mean, geometric mean & harmonic mean of $a_1, a_2, \ldots\ldots, a_n$. Find an expression for the geometric mean of $G_1, G_2, \ldots\ldots, G_n$ in terms of $A_1, A_2, \ldots\ldots, A_n, H_1, H_2, \ldots\ldots, H_n$.

15. Let a, b be positive real numbers. If a, A_1, A_2, b are in arithmetic progression, a, G_1, G_2, b are in geometric progression and a, H_1, H_2, b are in harmonic progression, show that

$$\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a+b)(a+2b)}{9ab}.$$

PART - II : OBJECTIVE QUESTIONS

Single choice type

1. If $x_i > 0$, i = 1, 2, ..., 50 and $x_1 + x_2 + .. + x_{50} = 50$, then the minimum value of

$\frac{1}{x_1} + \frac{1}{x_2} + \ldots.. + \frac{1}{x_{50}}$ equals to

(A) 50 (B) $(50)^2$ (C) $(50)^3$ (D) $(50)^4$

2. If a, $a_1, a_2, a_3, \ldots, a_{2n}$, b are in A.P. and a, $g_1, g_2, g_3, \ldots.. g_{2n}$, b are in G.P. and h is the harmonic mean of a and b, then $\frac{a_1 + a_{2n}}{g_1 g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2 g_{2n-1}} + \ldots + \frac{a_n + a_{n+1}}{g_n g_{n+1}}$ is equal to

(A) $\frac{2n}{h}$ (B) 2nh (C) nh (D) $\frac{n}{h}$

3. One side of an equilateral triangle is 24 cm. The mid–points of its sides are joined to form another triangle whose mid – points are in turn joined to form still another triangle. This process continues indefinitely. Then the sum of the perimeters of all the triangles is

(A) 144 cm (B) 212 cm (C) 288 cm (D) none of these

4. If the sum of n terms of a G.P. (with common ratio r) beginning with the p^{th} term is k times the sum of an equal number of terms of the same series beginning with the q^{th} term, then the value of k is:

(A) $r^{p/q}$ (B) $r^{q/p}$ (C) r^{p-q} (D) r^{p+q}

5. If P, Q be the A.M., G.M. respectively between any two rational numbers a and b, then P – Q is equal to

(A) $\frac{a-b}{a}$ (B) $\frac{a+b}{2}$ (C) $\frac{2ab}{a+b}$ (D) $\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{2}}\right)^2$

6. In a G.P. of positive terms, any term is equal to the sum of the next two terms. The common ratio of the G.P. is

(A) 2 cos 18° (B) sin 18° (C) cos 18° (D) 2 sin 18°

7. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots$ upto $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots =$

(A) $\pi^2/12$ (B) $\pi^2/24$ (C) $\pi^2/8$ (D) none of these

8. If $a_1, a_2, \ldots a_n$ are in A.P. with common difference $d \neq 0$, then the sum of the series

$(\sin d)\,[\operatorname{cosec} a_1 \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \ldots + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n]$

(A) $\sec a_1 - \sec a_n$ (B) $\operatorname{cosec} a_1 - \operatorname{cosec} a_n$

(C) $\cot a_1 - \cot a_n$ (D) $\tan a_1 - \tan a_n$

9. Sum of the series
$S = 1^2 - 2^2 + 3^2 - 4^2 + ... - 2002^2 + 2003^2$ is
(A) 2007006 (B) 1005004 (C) 2000506 (D) none of these

10. If $H_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$, then value of $1 + \frac{3}{2} + \frac{5}{3} + ... + \frac{2n-1}{n}$ is
(A) $2n - H_n$ (B) $2n + H_n$ (C) $H_n - 2n$ (D) $H_n + n$

11. If S_1, S_2, S_3 are the sums of first n natural numbers, their squares, their cubes respectively, then $\frac{S_3(1+8S_1)}{S_2^2}$ is equal to
(A) 1 (B) 3 (C) 9 (D) 10

12. Consider the following statements :
S_1 : Equal numbers are always in A.P. , G.P. and H.P.
S_2 : If x > 1 and $\left(\frac{1}{x}\right)^a, \left(\frac{1}{x}\right)^b, \left(\frac{1}{x}\right)^c$ are in G.P., then a, b, c are in A.P.
S_3 : If a, b, c be in H.P., then $a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$ will be in AP.
S_4 : If G_1 and G_2 are two geometric means and A is the arithmetic mean inserted between two positive numbers, then the value of $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$ is 2A.
State, in order, whether S_1, S_2, S_3, S_4 are true or false
(A) FTFT (B) TTTT (C) FFFF (D) TFTF

More than one choice type

13. The sides of a right triangle form a G.P. The tangent of the smallest angle is
(A) $\sqrt{\frac{\sqrt{5}+1}{2}}$ (B) $\sqrt{\frac{\sqrt{5}-1}{2}}$ (C) $\sqrt{\frac{2}{\sqrt{5}+1}}$ (D) $\sqrt{\frac{2}{\sqrt{5}-1}}$

14. If b_1, b_2, b_3 ($b_i > 0$) are three successive terms of a G.P. with common ratio r, the value of r for which the inequality $b_3 > 4b_2 - 3b_1$ holds is given by
(A) r > 3 (B) 0 < r < 1 (C) r = 3.5 (D) r = 5.2

PART - III : MATCH THE COLUMN

1.

Column – I	Column – II
(A) If $\log_5 2$, $\log_5(2^x - 5)$ and $\log_5(2^x - 7/2)$ are in A.P., then value of 2x is equal to	(p) 6
(B) Let S_n denote sum of first n terms of an A.P. If $S_{2n} = 3S_n$, then $\frac{S_{3n}}{S_n}$ is	(q) 9
(C) Sum of infinite series $4 + \frac{8}{3} + \frac{12}{3^2} + \frac{16}{3^3} + - - -$ is	(r) 3
(D) The length,breadth, height of a rectangular box are in G.P. The volume is 27, the total surface area is 78. Then the length is	(s) 1

2. **Column – I** | **Column – II**

	Column – I		Column – II
(A)	If $\log_x y$, $\log_z x$, $\log_y z$ are in G.P., xyz = 64 and x^3, y^3, z^3 are in A.P., then $\frac{3x}{y}$ is equal to	(p)	2
(B)	The value of $2^{\frac{1}{4}} . 4^{\frac{1}{8}} . 8^{\frac{1}{16}}$ - - - ∞ is equal to	(q)	1
(C)	If x, y, z are in A.P., then (x + 2y – z) (2y + z – x) (z + x – y) = kxyz, where k ∈ N, then k is equal to	(r)	3
(D)	There are m A.M. between 1 and 31. If the ratio of the 7^{th} and $(m-1)^{th}$ means is 5 : 9, then $\frac{m}{7}$ is equal to	(s)	4

PART - IV : COMPREHENSIONS

Comprehension # 1

We know that $1 + 2 + 3 + ... = \frac{n(n+1)}{2} = f(n)$,

$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6} = g(n)$,

$1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2 = h(n)$

1. g(n) – g(n – 1) must be equal to
(A) n^2 (B) $(n-1)^2$ (C) n – 1 (D) n^3

2. Greatest even natural number which divides g(n) – f(n), for every n ≥ 2, is
(A) 2 (B) 4 (C) 6 (D) none of these

3. f(n) + 3 g(n) + h(n) is divisible by 1 + 2 + 3 + ... + n
(A) only if n = 1 (B) only if n is odd (C) only if n is even (D) for all n ∈ N

Comprehension # 2

In a sequence of (4n + 1) terms the first (2n + 1) terms are in AP whose common difference is 2, and the last (2n + 1) terms are in GP whose common ratio 0.5. If the middle terms of the AP and GP are equal, then

4. Middle term of the sequence is
(A) $\frac{n . 2^{n+1}}{2^n - 1}$ (B) $\frac{n . 2^{n+1}}{2^{2n} - 1}$ (C) n . 2^n (D) None of these

5. First term of the sequence is
(A) $\frac{4n + 2n . 2^n}{2^n - 1}$ (B) $\frac{4n - 2n . 2^n}{2^n - 1}$ (C) $\frac{2n - n . 2^n}{2^n - 1}$ (D) $\frac{2n + n . 2^n}{2^n - 1}$

6. Middle term of the GP is
(A) $\frac{2^n}{2^n - 1}$ (B) $\frac{n . 2^n}{2^n - 1}$ (C) $\frac{n}{2^n - 1}$ (D) $\frac{2n}{2^n - 1}$

Exercise # 3

PART - I : IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. If $\alpha \in \left(0, \frac{\pi}{2}\right)$, then $\sqrt{x^2 + x} + \frac{\tan^2\alpha}{\sqrt{x^2 + x}}$ is always greater than or equal to:

[IIT-JEE-2003, Scr. , (3, –1), 84]

(A) $2 \tan \alpha$ (B) 1 (C) 2 (D) $\sec^2 \alpha$

2. If a, b & c are in arithmetic progression and a^2, b^2 & c^2 are in harmonic progression, then prove that either a = b = c or a, b & $-\frac{c}{2}$ are in geometric progression. **[IIT-JEE-2003, Main, (4, 0), 60]**

3. An infinite G.P. has first term as x and sum upto infinity as 5. Then the range of values of 'x' is:

[IIT-JEE-2004, Scr. , (3, –1), 84]

(A) $x \le -10$ (B) $x \ge 10$ (C) $0 < x < 10$ (D) $-10 \le x \le 10$

4. In the quadratic equation $ax^2 + bx + c = 0$, $a \ne 0$, $\Delta = b^2 - 4ac$ and $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in G.P. where α, β are the root of $ax^2 + bx + c = 0$, then **[IIT-JEE-2005, Scr. , (3, –1), 84]**

(A) $\Delta \ne 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $\Delta = 0$

5. If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where n > 1 and the runs scored in the k^{th} match are given by k. 2^{n+1-k}, where $1 \le k \le n$, find n **[IIT-JEE-2005,Main, (2, 0), 60]**

6. If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + ...(-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$, then find the minimum natural number n_0 such that $b_n > a_n \ \forall \ n > n_0$ **[IIT-JEE 2006, (6, 0), 184]**

Comprehension # 1

Let V_r denotes the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is (2r – 1). Let

$T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2, ... **[IIT-JEE 2007, Paper-1, (4, –1), 81]**

7. The sum $V_1 + V_2 + ... + V_n$ is

(A) $\frac{1}{12}$ n(n + 1) (3n² – n + 1) (B) $\frac{1}{12}$ n(n + 1) (3n² + n + 2)

(C) $\frac{1}{2}$ n(2n² – n + 1) (D) $\frac{1}{3}$ (2n³ – 2n + 3)

8. T_r is always

(A) an odd number (B) an even number (C) a prime number (D) a composite number

9. Which one of the following is a correct statement ?

(A) Q_1, Q_2, Q_3,...... are in A.P. with common difference 5

(B) Q_1, Q_2, Q_3,...... are in A.P. with common difference 6

(C) Q_1, Q_2, Q_3,..... are in A.P. with common difference 11

(D) $Q_1 = Q_2 = Q_3 = ...$

Comprehension # 2

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively. **[IIT-JEE 2007, Paper-2, (4, –1), 81]**

10. Which one of the following statements is correct ?

(A) $G_1 > G_2 > G_3 > ....$
(B) $G_1 < G_2 < G_3 < ......$
(C) $G_1 = G_2 = G_3 = ...$
(D) $G_1 < G_3 < G_5 < ......$ and $G_2 > G_4 > G_6 > ....$

11. Which one of the following statements is correct ?

(A) $A_1 > A_2 > A_3 > ......$
(B) $A_1 < A_2 < A_3 < ......$
(C) $A_1 > A_3 > A_5 > ....$ and $A_2 < A_4 < A_6 < ....$
(D) $A_1 < A_3 < A_5 < ....$ and $A_2 > A_4 > A_6 > ....$

12. Which one of the following statements is correct ?

(A) $H_1 > H_2 > H_3 > .....$
(B) $H_1 < H_2 < H_3 < ......$
(C) $H_1 > H_3 > H_5 > ...$ and $H_2 < H_4 < H_6 < ......$
(D) $H_1 < H_3 < H_5 < ....$ and $H_2 > H_4 > H_6 > ......$

13. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$

STATEMENT -1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.
STATEMENT-2 : The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True **[IIT-JEE 2008, Paper-2, (3, –1), 81]**

14. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is **[IIT-JEE - 2009, Paper-2, (3, –1), 80]**

(A) $\dfrac{n(4n^2-1)c^2}{6}$ (B) $\dfrac{n(4n^2+1)c^2}{3}$ (C) $\dfrac{n(4n^2-1)c^2}{3}$ (D) $\dfrac{n(4n^2+1)c^2}{6}$

15*. For $0 < \theta < \dfrac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \dfrac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \dfrac{m\pi}{4}\right) = 4\sqrt{2}$ is(are)

[IIT-JEE - 2009, Paper-2, (4, –1), 80]

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{6}$ (C) $\dfrac{\pi}{12}$ (D) $\dfrac{5\pi}{12}$

16. Let S_k, k = 1, 2,...., 100, denote the sum of the infinite geometric series whose first term is $\dfrac{k-1}{k!}$ and the common ratio is $\dfrac{1}{k}$. Then the value of $\dfrac{100^2}{100!} + \sum_{k=1}^{100} \left|(k^2 - 3k + 1)S_k\right|$ is

[IIT-JEE - 2010, Paper-1, (3, 0), 84]

17. Let $a_1, a_2, a_3, ..., a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, ..., 11.

If $\dfrac{a_1^2 + a_2^2 + ... + a_{11}^2}{11} = 90$, then the value of $\dfrac{a_1 + a_2 + ... + a_{11}}{11}$ is equal to

[IIT-JEE - 2010, Paper-2, (3, 0), 79]

18. Let $a_1, a_2, a_3,....., a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \leq p \leq 100$.

For any integer n with $1 \leq n \leq 20$, let m = 5n. If $\dfrac{S_m}{S_n}$ does not depend on n, then a_2 is

[IIT-JEE 2011, Paper-1, (4, 0), 80]

19. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} where $a > 0$ is **[IIT-JEE 2011, Paper-1, (4, 0), 80]**

20. Let $a_1, a_2, a_3, ...$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is **[IIT-JEE 2012, Paper-2, (3, –1), 66]**
(A) 22 (B) 23 (C) 24 (D) 25

PART - II : AIEEE PROBLEMS (PREVIOUS YEARS)

1. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in GP with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3): **[AIEEE 2003]**
(1) lie on a straight line (2) lie on an elipse
(3) lie on a circle (4) are vertices of a triangle.

2. Let T_r be the rth term of an AP whose first term is a and common difference is d. If for some positive integers m & n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals : **[AIEEE 2004]**
(1) 0 (2) 1 (3) $\frac{1}{mn}$ (4) $\frac{1}{m}+\frac{1}{n}$

3. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a,b,c are in AP and $|a| < 1$, $|b| < 1$, $|c| < 1$, then x,y,z are in :
(1) HP (2) Arithmetico–Geometric Progression **[AIEEE 2005]**
(3) AP (4) GP

4. If in a ΔABC, the altiudes from the vertices A, B, C on opposite sides are in H.P., then sin A, sin B, sin C are in-
(1) G.P. (2) A.P. **[AIEEE 2005]**
(3) Arithmetico-Geometirc progression (4) H.P.

5. Let $a_1, a_2, a_3, ...$ be terms of an AP. If $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals : **[AIEEE 2006]**
(1) $\frac{7}{2}$ (2) $\frac{2}{7}$ (3) $\frac{11}{41}$ (4) $\frac{41}{11}$

6. If $a_1, a_2, ..., a_n$ are in HP, then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to : **[AIEEE 2006]**
(1) $(n – 1)(a_1 – a_n)$ (2) na_1a_n (3) $(n – 1)a_1a_n$ (4) $n(a_1 – a_n)$

7. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals **[AIEEE 2007]**
(1) $\frac{1}{2}(1-\sqrt{5})$ (2) $\frac{1}{2}\sqrt{5}$ (3) $\sqrt{5}$ (4) $\frac{1}{2}(\sqrt{5}-1)$

8. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference –2, then the time taken by him to count all notes is **[AIEEE 2010]**
(1) 34 minutes (2) 125 minutes (3) 135 minutes (4) 24 minutes

9. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after : **[AIEEE 2011]**
(1) 18 months (2) 19 months (3) 20 months (4) 21 months

10. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is :
(1) $\alpha - \beta$ (2) $\frac{\alpha-\beta}{100}$ (3) $\beta - \alpha$ (4) $\frac{\alpha-\beta}{200}$ **[AIEEE 2011]**

11. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is **[AIEEE - 2013, (4, –¼),360]**
(1) $\frac{7}{81}(179 - 10^{-20})$ (2) $\frac{7}{9}(99 - 10^{-20})$ (3) $\frac{7}{81}(179 + 10^{-20})$ (4) $\frac{7}{9}(99 + 10^{-20})$

Answers

BOARD LEVEL SOLUTIONS

1. Given, $t_1 = 1, t_2 = 2, t_{n+2} = t_n + t_{n+1}$
Putting n = 1, we get $t_3 = t_1 + t_2 = 1 + 2 = 3$
n = 2, we get $t_4 = t_2 + t_3 = 2 + 3 = 5$
n = 3, we get $t_5 = t_3 + t_4 = 3 + 5 = 8$
Thus the first five terms of the given sequence are 1, 2, 3, 5 and 8.

2. Let the number of terms be n
given $t_n = 100, a = 20, d = 5$, we have to find n.
Now $t_n = a + (n – 1)d$
$\therefore$ 100 = 20 + (n – 1)5 or 80 = (n – 1) 5
or n – 1 = 16
$\therefore$ n = 17

3. If possible let nth term of the sequence be 55.
Now $t_n = a + (n – 1)d$
Here $t_n = 55, a = 1, d = 2$
$\therefore$ 55 = 1 + (n – 1)2 or 2n = 56
$\therefore$ n = 28
Hence 55 is 28th term of the given sequence

Note : If n does not come out to be an integer, then 55 will not be a term of the given sequence.

4. Terms of the given series are in A.P. whose common difference d = – 4 and first term a = 99
Now sum of 20 terms of the series

$$S_{20} = \frac{20}{2}[2.99 + (20 – 1)(– 4)]$$
$$= 10 (198 – 76) = 1220$$

5. Given sum of n term of any sequence = $n^2 + 2n$
we know $S_n = t_1 + t_2 + t_3 + ... t_n = n^2 + 2n$
Put n = 1, $S_1 = t_1 = 1 + 2 = 3$
Put n = 2, $S_2 = t_1 + t_2 = (2)^2 + 2(2) = 8$
Put n = 3, $S_3 = t_1 + t_2 + t_3 = 3^2 + 2 \times 3 = 15$
$\therefore$ $S_2 – S_1 = t_2 = 5$
$S_3 – S_2 = t_3 = 7$
Hence sequence is 3, 5, 7, ... which is A.P. whose first term is 3 and common difference is 2.

Note : If general term of any sequence is linear expression of n. ($t_n = an + b$) and sum of n terms is quadratic expression ($S_n = an^2 + bn + c$) then sequence is A.P.

6. Here a = 1, r = 2, n = 20, to find t_n
Now $t_n = ar^{n–1} = 1.(2)^{20–1} = 2^{19}$
Hence the boy will get 2^{19} rupees on 20th of April

7. Let the number of terms be n
Given, a = 5, r = 4, $t_n = 5120$
$\because$ $t_n = ar^{n–1}$ $\therefore$ $5120 = 5.4^{n–1}$
or $4^{n–1} = 1024 = 4^5$
$\therefore$ n – 1 = 5 $\Rightarrow$ n = 6

8. Given $t_7 = 8t_4$ $\therefore$ $ar^6 = 8ar^3$
where a and r are the first term and common ratio respectively of the G.P.
or $r^3 = 8 = 2^3$ $\therefore$ r = 2
Also $t_5 = 48$ $\therefore$ $ar^4 = 48$
or $a(2)^4 = 48$ or 16a = 48
$\therefore$ a = 3
Hence required G.P. is 3, 6, 12, 24, ...

9. Let the sum of n terms of the given series be 3280

$$\therefore S_n = \frac{a(1-r^n)}{1-r} \qquad \therefore 3280 = \frac{1.(1-3^n)}{1-3}$$

or $\frac{3^n-1}{3-1} = 3280$
or $3^n – 1 = 6560$
or $3^n = 6561 = 3^8$ $\therefore$ n = 8

10. Here terms of given series are in G.P. and

$$a = 1, r = \frac{x}{1+x} \text{ Also } |r| = \left|\frac{x}{1+x}\right| < 1$$

Now $S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{x}{1+x}} = 1 + x$

11. Let $S = 4^3 + 5^3 + 6^3 + ... 10^3$
$= (1^3 + 2^3 + 3^3 + 4^3 + ... 10^3) – (1^3 + 2^3 + 3^3)$

$$= \left(\frac{10(10+1)}{2}\right)^2 - \left(\frac{3(3+1)}{2}\right)^2 = 55^2 – 6^2 = 2989$$

12. If $0 < \theta < \frac{\pi}{2}$, then $\tan\theta$, $\cot\theta$ both are positive number
$\therefore$ A.M. $\geq$ G. M.

$$\frac{\tan\theta + \cot\theta}{2} \geq (\tan\theta \cdot \cot\theta)^{1/2}$$

$\Rightarrow$ $\tan\theta + \cot\theta \geq 2$

13. nth terms of A.P. whose first terms is a and common difference is d is given by
$t_n = a + (n – 1)d$
Given when n = 7, $t_n = 34$
$\therefore$ 34 = a + 6d ...(i)
when n = 13, $t_n = 64$
$\therefore$ 64 = a + 12d ...(ii)
subtracting (i) from (ii) we get 30 = 6d
$\therefore$ d = 5
Putting d = 5 in (i), we get 34 = a + 30
$\therefore$ a = 4
Hence the required A.P. is 4, 9, 14, 19, 24, ...

14. First even number between 101 and 999 is 102 and the last even number is 998 and difference between two consecutive even number is 2.
Hence a = 102, d = 2, $t_n = 998$
$\because$ $t_n = a + (n – 1) d$
$\therefore$ 998 = 102 + (n – 1)2
or 2(n – 1) = 896

or $n - 1 = 448$

$\therefore$ $n = 449$

Now sum of all even numbers between 101 to 999

$= \frac{n}{2}$ (First term + Last term)

$= \frac{449}{2}(102 + 998) = 246950$

15. Let G_1, G_2, G_3, G_4 be the four G.M's between 5 and 160

$\therefore$ $5, G_1, G_2, G_3, G_4, 160$ will be in G.P.

Now $160 = $ 6th term of G.P. $= ar^5 = 5r^5$ $(\because a = 5)$

or $r^5 = 32 = 2^5$ $\therefore r = 2$

Now $G_1 = 5r = 10$

$G_2 = 5r^2 = 20$

$G_3 = 5r^3 = 40$

$G_4 = 5r^4 = 80$

16. Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ be the seven A.M.'s between 2 and 34

$\therefore$ $2, A_1, A_2, A_3, A_4, A_5, A_6, A_7, 34$ wil be in A.P.

Now $34 = $ 9th term of A.P. $= a + 8d = 2 + 8d$ $[\because a = 2]$

or $8d = 32$ $\therefore d = 4$

Now $A_1 = a + d = 2 + 4 = 6$

$A_2 = a + 2d = 2 + 2\cdot4 = 10$

$A_3 = a + 3d = 2 + 3\cdot4 = 14$

$A_4 = a + 4d = 2 + 4\cdot4 = 18$

$A_5 = a + 5d = 2 + 5\cdot4 = 22$

$A_6 = a + 6d = 2 + 6\cdot4 = 26$

$A_7 = a + 7d = 2 + 7\cdot4 = 30$

17. Let a be the first term and d the common difference of A.P.

Given that $nt_n = mt_m$

$\therefore$ $n[a + (n - 1)d] = m[a + (m - 1)d]$

or $(m - n)a = d[n(n - 1) - m(m - 1)]$

or $(m - n)a = d[(m - n) - (m^2 - n^2)]$

or $(m - n)a = d(m - n)[1 - (m + n)]$

or $a = d(1 - m - n) = [1 - (m + n)]$ $[\because m \neq n]$

or $-a = d[m + n - 1]$ ...(1)

Now $(m + n)^{th}$ term

$t_{m+n} = a + (m + n - 1)d = a - a = 0$ [from (i)]

18. Let the three numbers in A.P. be $a - d, a, a + d$

Given $(a - d) + a + (a + d) = 27$ or $3a = 27$ $\therefore a = 9$

and $(a - d)^2 + a^2 + (a + d)^2 = 275$

or $a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 275$

or $3a^2 + 2d^2 = 275$ or $3(9)^2 + 2d^2 = 275$

or $2d^2 = 275 - 243 = 32$ or $d^2 = 16$ $\therefore d = \pm 4$

If d = 4, the three number are 5, 9, 14

If d = – 4 the three numbers are 14, 9, 5

19. a, b, c are in A.P.

$\Rightarrow$ $a - (a + b + c), b - (a + b + c), c - (a + b + c)$ are in A.P.

$\Rightarrow$ $-(b + c), -(a + c), -(a + b)$ are in A.P.

$\Rightarrow$ $b + c, c + a, a + b$ are in A.P.

20. given a, b, c are in A.P. Let d be common difference

Then $b = a + d, c = a + 2d$

Now $2(b - c) = 2(a + d - a - 2d) = -2d$,

$a - c = a - (a + 2d) = -2d$

Hence $2(a - b) = a - c = 2(b - c)$

(ii) $(a - c)^2 = (a - a - 2d)^2 = 4d^2$

$4(b^2 - ac) = 4[(a + d)^2 - a(a + 2d)]$

$= 4[a^2 + 2ad + d^2 - a^2 - 2ad] = 4d^2$

Hence $(a - c)^2 = 4(b^2 - ac)$

21. Terms of given series are in A.P. Whose first term $= (a + b)^2$

and common difference $= (a^2 + b^2) - (a + b)^2 = -2ab$

Now sum of n terms of given series

$$S_n = \frac{n}{2}[2(a + b)^2 + (n - 1)(-2ab)]$$

$$= \frac{n}{2} 2[(a^2 + b^2) + 2ab - (n - 1)ab]$$

$$= n(a^2 + b^2) + nab(3 - n)$$

22. Given $1 + 3 + 5 + 7 + ... +$ to n terms ≥ 500

or $\frac{n}{2}[2(1) + (n - 1)(2)] \geq 500$ or $n^2 \geq 500$

$\therefore$ $n \geq \sqrt{500}$ or $n \leq -\sqrt{500}$

But n is a positive integer

$\therefore$ $n \geq \sqrt{500}$ or $n \geq 22.36$

$\therefore$ least value of n = 23.

23. Let $S_n = 8 + 88 + 888 + ...$ to n terms

$= 8[1 + 11 + 111 + ...$ to n terms]

$$= \frac{8}{9}[9 + 99 + 999 + ... + \text{ to n terms}]$$

$$= \frac{8}{9}[(10 - 1) + (10^2 - 1) + (10^3 - 1) + ... \text{ n terms}]$$

$$= \frac{8}{9}[(10 + 10^2 + 10^3 + ... 10^n] - (1 + 1 + 1 .... \text{ n terms})$$

$$= \frac{8}{9}\left[10\frac{10^n - 1}{10 - 1} - n\right] = \frac{8}{81}[10^{n+1} - 10 - 9n]$$

24. Let $x = 0.54 = 0.545454 ....$ to ∞

$= 0.54 + 0.0054 + 0.000054 + ...$ to ∞

$$= \frac{54}{10^2} + \frac{54}{10^4} + \frac{54}{10^6} + ... \text{ to } \infty$$

$$= 54\left[\frac{1}{10^2} + \frac{1}{10^4} + \frac{1}{10^6} + ... \text{to } \infty\right]$$

$$= 54\left(\frac{\frac{1}{10^2}}{1 - \frac{1}{10^2}}\right) = 54 \times \frac{1}{100 - 1} = \frac{54}{99}$$

25. Let first terms of infinite G.P. is a.

then $a, ar, ar^2, ar^3 ....$ to ∞

Given $\frac{t_n}{S_\infty - S_n} = \frac{ar^{n-1}}{\frac{a}{1-r} - \frac{a(1-r^n)}{(1-r)}}$

$= \frac{ar^{n-1}}{\frac{a}{1-r}(1-(1-r^n))} = \frac{ar^{n-1}}{\frac{a}{(1-r)}.r^n} = \frac{1-r}{r}$

26. $\sum_{r=1}^{n}(3^r - 2^r) = \sum_{r=1}^{n}3^r - \sum_{r=1}^{n}2^r$

$= (3^1 + 3^2 + ... + 3^n) - (2^1 + 2^2 + ... + 2^n)$

$= \frac{3(3^n - 1)}{(3-1)} - \frac{2(2^n - 1)}{(2-1)}$

$= \frac{3^{n+1}}{2} - \frac{3}{2} - 2^{n+1} + 2$

$= \frac{3^{n+1}}{2} - 2^{n+1} + \frac{1}{2} = \frac{3^{n+1} - 2^{n+2} + 1}{2}$

27. Let $S = \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...$ to n term

$= \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + \left(1 - \frac{1}{16}\right) + ...$ n term

= (1 + 1 + 1 + ... n times)

$- \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...\text{n term}\right)$

$= n - \frac{\frac{1}{2}\left(1 - \left(\frac{1}{2}\right)^n\right)}{\left(1 - \frac{1}{2}\right)} = n - 1 + \frac{1}{2^n} = \frac{2^{n+1} - 2^n + 1}{2^n}$

28. Let the A.P. be a, a + d, a + 2d, a + 3d,.......a + 2nd
sum of its odd terms
= a + (a + 2d) + (a + 4d) + ... to (n + 1) terms

$= \frac{n+1}{2}$ [2a + (n + 1 – 1) 2d] = (n + 1) (a + nd)

sum of even terms = (a + d) + (a + 3d) + ... to n terms

$= \frac{n}{2}$ [2(a + d) + (n – 1) 2d] = n(a + nd)

$\frac{\text{sum of odd terms}}{\text{sum of even terms}} = \frac{n+1}{n}$

29. Given a, b, c are in A.P. $\therefore$ b – a = c– b
= common difference (Let d)
$\therefore$ (c – b) + (b – a) = 2d or c – a = 2d

and x, y, z are in G.P. $\frac{y}{x} = \frac{z}{y}$ = r (common ratio of G.P.)

$\therefore$ y = xr, z = yr = xr^2

Now $x^{b-c} \cdot y^{c-a} . z^{a-b} = x^{b-c} (xr)^{c-a} (xr^2)^{a-b}$

$= (x)^{-d} (xr)^{2d} (xr^2)^{-d}$

$= x^{-d+2d-d} (r)^{2d-2d}$

$= x^0 . x^0$

= 1

30. Let a, b be the two numbers
Given A.M. between a and b = 5

$\therefore \frac{a+b}{2} = 5 \Rightarrow a + b = 10$ ...(i)

$\because$ 3 is G.M. between a and b
$\therefore$ a, 3, b will be in G.P.
$3^2 = ab \Rightarrow ab = 9$ ...(ii)
Put value of b from (ii) into (i)

$a + \frac{9}{a} = 10 \Rightarrow a^2 - 10a + 9 = 0$

$\therefore$ a = 1, 9
When a = 1, b = 9
When a = 9, b = 1
Thus the numbers are 1 and 9 or 9 and 1

31. We have 1111.....1 (91 digits)
$= 10^{90} + 10^{89} + ... + 10^2 + 10^1 + 10^0$

$= \frac{10^{91} - 1}{10 - 1} = \frac{(10^{91} - 1)}{10 - 1} \times \left(\frac{10^7 - 1}{10^7 - 1}\right)$

$= \frac{10^{91} - 1}{10^7 - 1}\left(\frac{10^7 - 1}{10 - 1}\right)$

$= (10^{84} + 10^{77} + 10^{70} + ... + 1)(10^6 + 10^5 + ... + 1)$
Thus 111.... 1 (91 digits) is not a prime number

32. Let $t_n = 12n^2 - 6n + 5$

$\therefore S_n = \sum_{n=1}^{n} t_n = \sum_{n=1}^{n} 12n^2 - 6n + 5$

$= \sum_{n=1}^{n} 12n^2 - \sum_{n=1}^{n} 6n + \sum_{n=1}^{n} 5$

$= 12\sum_{n=1}^{n} n^2 - 6\sum_{n=1}^{n} n + 5n$

$= 12\frac{n(n+1)(2n+1)}{6} - 6\frac{n(n+1)}{2} + 5n$

= 2n(n + 1) (2n + 1) – 3n(n + 1) + 5n

$= 4n^3 + 3n^2 + 4n.$

33. Let the three numbers in G.P. is $\frac{a}{r}$, a, ar

Given $\frac{a}{r}$. a . ar = 216

$\Rightarrow a^3 = 216$

$\therefore$ a = 6

Also $\frac{a}{r} . a + a . ar + ar . \frac{a}{r} = 156$

or $a^2\left(\frac{1}{r}+r+1\right) = 156$ or $\frac{r^2+r+1}{r} = \frac{13}{3}$

or $3r^2 - 10r + 3 = 0 \quad \Rightarrow r = 3$ or $\frac{1}{3}$

If $r = \frac{1}{3}$, then the number are 18, 6, 2

If r = 3, then the number are 18, 6, 2

34. Given $a^{1/x} = b^{1/y} = c^{1/z} = \lambda$ (Let)
then $a = \lambda^x, b = \lambda^y, c = \lambda^z$
$\therefore$ a, b, c are in G.P., then
$b^2 = ac$
$\lambda^{2y} = \lambda^x . \lambda^z$
$\lambda^{2y} = \lambda^{x+z}$
$\therefore$ 2y = x + z
Hence x, y, z are in A.P.

35. Given a, b, c, d are in G.P.

For first three terms A.M. > G.M.

$\frac{a+c}{2} > b$ ....(i)

For last three terms $\frac{b+d}{2} > c$ ...(ii)

add (i) + (ii), we get $\frac{a+b+c+d}{2} > b + c$

$\therefore$ a + d > b + c

36. Here $S_{10} = 140$, $S_{16} = 320$, to find S_n Now

$140 = S_{10} = \frac{10}{2}[2a + (10 - 1) d]$

or 140 = 5(2a + 9d) or 2a + 9d = 28 ...(i)

and $320 = S_{16} = \frac{16}{2}[2a + (16 - 1)d]$

or 2a + 15d = 40 ...(ii)
subtrating (ii) – (i) $\Rightarrow$ 6d = 12 $\therefore$ d = 2
Put d = 2 in (i) we get a = 5

Now $S_n = \frac{n}{2}[2 \times 5 + (n - 1) 2] = n^2 + 4n$

37. Let a is first term and d is common difference of A.P.
Then $\ell = a + (n - 1)d$ ....(i)
where n is number of terms in A.P.

and $S = \frac{n}{2}(a + \ell)$ ...(ii) $\therefore n = \frac{2S}{a+\ell}$

Put value of n in (i) $\ell = a + \left(\frac{2S}{a+\ell} - 1\right)d$

or $d = \frac{\ell - a}{\left(\frac{2S}{a+\ell} - 1\right)} = \frac{\ell^2 - a^2}{2S - (a+\ell)}$

38. Let the four number are a – 3d, a – d, a + d, a + 3d
Given, (a – 3d) + (a – d) + (a + d) + (a + 3d) = 24
or 4a = 24 $\therefore$ a = 6
and (a – 3d) (a – d) (a + d) (a + 3d) = 945
or $(a^2 - 9d^2)(a^2 - d^2) = 945$
or $(36 - 9d^2)(36 - d^2) = 945$
or $d^4 - 40d^2 + 144 = 105$
or $d^4 - 40d^2 + 39 = 0$
or $d^4 - d^2 - 39d^2 + 9 = 0$
or $(d^2 - 1)(d^2 - 39) = 0$
Since number are integers
$\therefore d^2 \neq 39$
$\therefore d^2 = 1 \quad \therefore d = \pm 1$
Hence Four integers are 3, 5, 7, 9 or 9, 7, 5, 3

39. Let the last three numbers in A.P. be $\alpha, \alpha + 6, \alpha + 12$
and the first number be a.
Hence the four numbers are a, $\alpha, \alpha + 6, \alpha + 12$
Given $a = (\alpha + 12)$ ...(i)
and a, $\alpha, \alpha + 6$ are in G.P. $\alpha^2 = a(\alpha + 6)$
or $\alpha^2 = (\alpha + 12)(\alpha + 6)$ [$\because a = \alpha + 12$]
or $18\alpha = -72 \quad \therefore \alpha = -4$
From (i) a = – 4 + 12 = 8
Hence the four numbrs are 8, – 4, 2 and 8

40. $S_1 = \frac{n}{2}[2(1) + (n - 1)(1)] = \frac{n(n+1)}{2}.1$

$S_2 = \frac{n}{2}[2(2) + (n - 1)(2)] = \frac{n(n+1)}{2}.2$

$S_3 = \frac{n}{2}[2(3) + (n - 1)(3)] = \frac{n(n+1)}{2}.3$

....
....

$S_P = \frac{n}{2}[2P + (n - 1)P] = \frac{n(n+1)}{2}P$

Now $S_1 + S_2 + S_3 + ... + S_P$

$= \frac{n(n+1)}{2}[1 + 2 + 3 + ...$ to P terms$]$

$= \frac{n(n+1)}{2}\frac{P(P+1)}{2} = \frac{np}{4}(n + 1)(P + 1)$

41. Let the three digits be a, ar and ar^2
Given $a + ar^2 = 2ar + 1$
or $a(r^2 - 2r + 1)$ or $a(r - 1)^2 = 1$

Also according to question, $a + ar = \frac{2}{3}(ar + ar^2)$

Or 3a (1 + r) = 2ar (1 + r) or (1 + r) (3 – 2r) = 0

$\therefore r = \frac{3}{2}, -1$

When r = – 1, $a = \frac{1}{4}$ which is not possible, for a is an integer

Hence $a = 4, ar = 4.\frac{3}{2} = 6, ar^2 = \frac{4.9}{4} = 9$

$\therefore$ required number is 469

42. Let $S = 3 + 5 + 9 + 15 + 23 \ ...... + t_{n-1} + t_n$ ....(i)
Again $S = \ + 3 + 5 + 9 + 15 .... + ... + t_{n-1} + t_n$ ...(ii)
Subtract (ii) from (i)
$0 = 3 + [2 + 4 + 6 + 8 + ... \text{ to } (n-1) \text{ terms}] - t_n$

or $t_n = 3 + \frac{n-1}{2}[2 \times 2 + (n-2) \cdot 2]$
$= 3 + (n-1)\,n = n^2 - n + 3$
$\therefore \ t_{30} = (30)^2 - 30 + 3 = 873$

43. Given a = A.M. between b and c $= \frac{b+c}{2}$
$\because$ g_1 and g_2 are two G.M. between b and c
$\therefore$ b, g_1, g_2, c are in G.P. $g_1 = br$, $g_2 = br^2$, $c = br^3$

where r is common ratio of G.P.
Now $g_1^3 + g_2^3 = (br)^3 + (br^2)^3 = b^3r^3(1 + r^3)$

$= b^3 \cdot \frac{c}{b}\left(1+\frac{c}{b}\right)$ $\left[\because r^3 = \frac{c}{b}\right]$

$= b^2c\left(\frac{b+c}{b}\right) = bc\,(b + c) = bc(2a)$

$\left[\because c = \frac{b+c}{2}\right] = 2abc$

Thus $g_1^3 + g_2^3 = 2abc$

44. Since A.M. $\geq$ G.M.

$\therefore \ \frac{y+z}{2} \geq \sqrt{yz}$ ...(i)

$\frac{z+x}{2} \geq \sqrt{zx}$ ...(ii)

$\frac{x+y}{2} \geq \sqrt{xy}$ ...(iii)

Multiplying (i), (ii) and (iii), we get

$\frac{(x+y)(y+z)(z+x)}{8} \geq 8xyz$

or $(1 - x)(1 - y)(1 - z) \geq 8xyz$ ($\because x + y + z = 1$)

45. Taking A.M. and G.M. of number

$\frac{a}{2}, \frac{a}{2}, \frac{b}{3}, \frac{b}{3}, \frac{b}{3}, \frac{c}{2}, \frac{c}{2}$, we get
A.M. $\geq$ G.M.

$\frac{2.\frac{a}{2} + 3.\frac{b}{3} + 2.\frac{c}{2}}{7} \geq \left(\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2\right)^{1/7}$

or $\frac{3}{7} \geq \left(\frac{a^2b^3c^2}{2^2.3^3.2^2}\right)^{1/7}$ or $\frac{3^7}{7^7} \geq \frac{a^2b^3c^2}{2^4.3^3}$

or $a^2b^3c^2 \leq \frac{3^{10}.2^4}{7^7}$

$\therefore$ Greatest value of $a^2\,b^3\,c^2 = \frac{3^{10}.2^4}{7^7}$

46. Let the number be $a - d, a, a + d, a + 2d$
where $a, d \in I$, $d > 0$
Given $(a - d)^2 + a^2 + (a + d)^2 = (a + 2d)$
or $2d^2 - 2d + (3a^2 - a) = 0$

$\therefore \ d = \frac{2 + \sqrt{4 - 4(2)(3a^2 - a)}}{2.2}$

or $d = \frac{\left(1 \pm \sqrt{1 + 2a - 6a^2}\right)}{2}$

Since, d is positive integer
$1 + 2a - 6a^2 > 0$

or $a^2 - \frac{a}{3} - \frac{1}{6} < 0$

or $\left(a - \frac{1-\sqrt{7}}{6}\right)\left(a - \frac{1+\sqrt{7}}{6}\right) < 0$

or $\frac{1-\sqrt{7}}{6} < a < \frac{1+\sqrt{7}}{6}$
since a is integer a = 0

then $d = \frac{1}{2}[1 \pm 1] = 1$ or 0 since $d > 0$

$\therefore$ d = 1
Hence, then numbers are –1, 0, 1, 2

47. Let $S_n = 1 + 5 + 11 + 19 + ... t_{n-1} + t_n$ ...(i)
and $S_n = \ + 1 + 5 + 11 + ... t_{n-2} + t_{n-1} + t_n$ ...(ii)
Subtracting (ii) from (i), we get
$0 = 1 + [4 + 6 + 8 + ... \text{ to } (n-1) \text{ terms}] - t_n$
or $t_n = 1 + [4 + 6 + 8 + ... \text{ to } (n-1) \text{ terms}]$

$= 1 + \frac{(n-1)}{2} \cdot [2.4 + (n - 1 - 1)\,2]$

$= 1 + (n - 1)(n + 2)$
$= n^2 + n - 1$
$\therefore \ S_n = \Sigma t_n = \Sigma(n^2 + n - 1) = \Sigma n^2 + \Sigma n - \Sigma 1$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n+(n+1)}{2} - n$

$= \frac{n(n^2 + 3n - 1)}{3}$

48. Let r terms be identical
Now sequence of identical terms is 6, 12, 18
Its rth terms = 6 + (r – 1) 6 = 6r
100 th term of the sequence 2, 4, 6, 8 ....
= 2 + (100 – 1) (2) = 200
and 80 th term of the sequence 3, 6, 9, ...
= 3 + (80 – 1) (3) = 240
Since, last term i.e. r th term of the sequence of identical terms cannot be greater than 200

$\therefore \ 6r \leq 200$ or $r \leq \frac{200}{6}$ or $r \leq 33\frac{1}{3}$

$\therefore$ r = 33 Hence 33 terms are identical

49. Let A be the first term and D is common difference of an A.P.

Given $t_p = a \Rightarrow A + (p-1)D = a$ ...(i)

$t_q = b \Rightarrow A + (q-1)D = b$ ...(ii)

Subtracting (i) – (ii), we get $(p-q)D = a-b$

$\therefore \quad D = \frac{a-b}{p-q}$

Adding (i) and (ii) we get $2A + (p+q-2)D = a+b$

$2A + (p+q-1)D = a+b+D$

$2A + (p+q-1)D = a+b+\frac{a-b}{p-q}$ [from value of D]

Now $S_{p+q} = \frac{p+q}{2}[2A + (a+q-1)]$

$= \frac{p+q}{2}\left[a+b+\frac{a-b}{p-q}\right]$

50. Let the First installment be a and common difference of A.P. be d.

Given 3600 = sum of 40 terms = $\frac{40}{2}$ [2a + (40 – 1) d]

or 3600 = 20 (2a + 39d) or 2a + 39d = 180 ...(i)

After 30 installments one third of the debt is unpaid

Hence $\frac{3600}{3}$ = 1200 is unpaid and 2400 is paid

Now 2400 = $\frac{30}{2}$ (2a + (30 – 1)d) or 160 = 2a + 29d ...(ii)

Substracting (ii) from (i), we get 20 = 10d $\therefore$ d = 2

From (i), 180 = 2a + 39.2 or 2a = 180 – 78 = 102 $\therefore$ a = 51

Now value of the 8th installments = a + (8 – 1)d

= 51 + 7.2 = Rs. 65

= 51 + 7.2 = Rs. 65

51. Sum of latter half of 2n terms = $S_{2n} - S_n$

$= \frac{2n}{2}[2a + (2n-1)d] - \frac{n}{2}[2a + (n-1)d]$

Where a is the first term and d the common difference of A.P.

$= \frac{n}{2}[4a + 2(2n-1)d - 2a - (n-1)d]$

$= \frac{n}{2}[2a + (4n - 2 - n + 1)] = \frac{n}{2}[2a + (3n-1)d]$

$= \frac{1}{3} \cdot \frac{3n}{2}[2a + (3n-1)d] = \frac{1}{3}S_{3n}$

$= \frac{1}{3}$ sum of the first 3n terms

EXERCISE # 1

PART - I

Section (A) :

A-1. 2, 5, 8,..... **A-2.** 612 **A-3.** 11

A-4. 128, 771 **A-5.** 19668 **A-6.** 160

A-7. 4, 9, 14

Section (B) :

B-1. 128 **B-2.** 2, 6, 18

B-4. 6, –3, 3/2, ...

Section (C) :

C-1. $\frac{1}{11}$ **C-2.** (i) $4 - \frac{2+n}{2^{n-1}}$ (ii) $\frac{8}{3}$

C-3. $n.2^{n+2} - 2^{n+1} + 2$.

Section (D) :

D-1. a = 4, b = 8 **D-3.** $\frac{1}{201}$

Section (E) :

E-1. (i) $2^{n+2} - 3n - 4$ (ii) $\frac{1}{27}(10^{n+1} - 9n - 10)$

E-2. (i) $\frac{1}{12} - \frac{1}{4(2n+1)(2n+3)}$

(ii) $\frac{n}{10}(n+1)(n+2)(n+3)(2n+3)$

Section (F) :

F-1. 3, 7, 11 or 12, 7, 2 **F-2.** $\frac{q-r}{p-q}$

F-3. (i) $\frac{1}{6}n(n+1)(2n+7)$

(ii) $\frac{1}{2}(3^{n+1}+1) - 2^{n+1}$

PART - II

Section (A) :

A-1. (D) **A-2.** (D) **A-3.** (C)

A-4. (C) **A-5.** (B) **A-6.** (A)

A-7. (D) **A-8.** (B) **A-9.** (D)

A-10*. (BD) **A-11*.** (AB)

Section (B) :

B-1. (B) **B-2.** (B) **B-3.** (D)
B-4. (B) **B-5.** (D) **B-6.** (C)
B-7. (C) **B-8*.** (AC) **B-9*.** (BC)

Section (C) :

C-1. (D) **C-2.** (C) **C-3.** (A)

Section (D) :

D-1. (B) **D-2*.** (ABC) **D-3.** (A)
D-4. (A)

Section (E) :

E-1*. (ABCD)

Section (F) :

F-1. (C) **F-2.** (C) **F-3.** (C)
F-4. (B)

PART - III

1. (A) **2.** (A) **3.** (D) **4.** (A)
5. (B)

EXERCISE # 2

PART - I

1. (i) $2^{n-2}(2^n + 2^{n-1} - 1)$ (ii) $(n-1)^3 + n^3$

4. $\frac{65}{36}$ **5.** $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{\pi}{3}$ **6.** $2\pi R^2$; $4R^2$

7. A = 3; B = 8

11. (i) (1/5) n (n + 1) (n + 2) (n + 3) (n + 4)

(ii) $\frac{n(n+1)}{4(n+2)}$

12. (i) $\frac{25}{54}$ (ii) $\frac{n(n+1)}{2(n^2+n+1)}$; $s_\infty = \frac{1}{2}$

13. (3 + 6 + 12 +......) ; (2/3 + 25/3 + 625/6 +......) G.P.

(2 + 5 + 8 + ...) ; $\left(\frac{25}{2} + \frac{79}{6} + \frac{83}{6} + \dots\dots\dots\right)$ A.P.

14. $G = \prod_{K=1}^{n} (A_k H_k)^{\frac{1}{2n}}$

PART - II

1. (A) **2.** (A) **3.** (A) **4.** (C) **5.** (D) **6.** (D)

7. (C) **8.** (C) **9.** (A) **10.** (A) **11.** (C) **12.** (A)

13. (BC) **14.** (ABCD)

PART - III

1. (A) $\rightarrow$ (p), (B) $\rightarrow$ (p), (C) $\rightarrow$ (q), (D) $\rightarrow$ (q)

2. (A) $\rightarrow$ (r), (B) $\rightarrow$ (p), (C) $\rightarrow$ (s), (D) $\rightarrow$ (p)

PART - IV

1. (A) **2.** (A) **3.** (D) **4.** (A) **5.** (B) **6.** (D)

EXERCISE # 3

PART - I

1. (A) **3.** (C) **4.** (C) **5.** 7

6. minimum natural number $n_0 = 5$ **7.** (B) **8.** (D)

9. (B) **10.** (C) **11.** (A) **12.** (B) **13.** (C) **14.** (C)

15*. (CD) **16.** 3 **17.** 0

18. 3 or 9, both 3 and 9 (The common difference of the arithmatic progression can be either 0 or 6, and accordingly the second term can be either 3, or 9 ; thus the answers 3, or 9, or both 3 and 9 are acceptable.)

19. 8 **20.** (D)

PART - II

1. (1) **2.** (1) **3.** (1) **4.** (2) **5.** (3) **6.** (3)

7. (4) **8.** (1) **9.** (4) **10.** (2) **11.** (3)

Advanced Level Problems

PART - I : OBJECTIVE QUESTIONS

Single choice type

1. Let $\{a_n\}$ and $\{b_n\}$ are two sequences given by $a_n = (x)^{1/2^n} + (y)^{1/2^n}$ and $b_n = (x)^{1/2^n} - (y)^{1/2^n}$ for all $n \in N$. The value of $a_1a_2a_3........a_n$ is equal to

(A) $x - y$ (B) $\frac{x+y}{b_n}$ (C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

2. If $1^2 + 2^2 + 3^2 + ... + 2003^2 = (2003)(4007)(334)$ and
$(1)(2003) + (2)(2002) + (3)(2001) + ... + (2003)(1) = (2003)(334)(x)$., then x equals
(A) 2005 (B) 2004 (C) 2003 (D) 2001

3. If $x > 0$, and $\log_2 x + \log_2 \left(\sqrt{x}\right) + \log_2 \left(\sqrt[4]{x}\right) + \log_2 \left(\sqrt[8]{x}\right) + \log_2 \left(\sqrt[16]{x}\right) + ... = 4$, then x equals to
(A) 2 (B) 3 (C) 4 (D) 5

4. If $\sum_{r=1}^{n} t_r = \frac{1}{12} n(n+1)(n+2)$, then the value of $\sum_{r=1}^{n} \frac{1}{t_r}$ is

(A) $\frac{2n}{n+1}$ (B) $\frac{n}{(n+1)}$ (C) $\frac{4n}{n+1}$ (D) $\frac{3n}{n+1}$

5. If a, b, c are in A.P., p, q, r are in H.P. and ap, bq, cr are in G.P., then $\frac{p}{r} + \frac{r}{p}$ is equal to

(A) $\frac{a}{c} + \frac{c}{a}$ (B) $\frac{a}{c} - \frac{c}{a}$ (C) $\frac{b}{q} + \frac{q}{b}$ (D) $\frac{b}{q} - \frac{a}{p}$

6. The common difference 'd' of the A.P. in which $T_7 = 9$ and $T_1T_2T_7$ is least, is

(A) $\frac{33}{2}$ (B) $\frac{5}{4}$ (C) $\frac{33}{20}$ (D) none of these

7. The H.M. between two numbers is $\frac{16}{5}$, their A.M. is A and G.M. is G. If $2A + G^2 = 26$, then the numbers are
(A) 6, 8 (B) 4, 8 (C) 2, 8 (D) 1, 8

8. If 1, 2, 3 ... are first terms; 1, 3, 5 .... are common differences and S_1, S_2, S_3 .... are sums of n terms of given p AP's; then $S_1 + S_2 + S_3 + ... + S_p$ is equal to

(A) $\frac{np(np+1)}{2}$ (B) $\frac{n(np+1)}{2}$ (C) $\frac{np(p+1)}{2}$ (D) $\frac{np(np-1)}{2}$

9. If a and b are p^{th} and q^{th} terms of an AP, then the sum of its (p + q) terms is

(A) $\frac{p+q}{2}\left[a-b+\frac{a+b}{p-q}\right]$ (B) $\frac{p+q}{2}\left[a+b+\frac{a-b}{p-q}\right]$

(C) $\frac{p-q}{2}\left[a+b+\frac{a+b}{p+q}\right]$ (D) none of these

10. If $S_n = \sum_{r=1}^{n} t_r = \frac{1}{6}n(2n^2 + 9n + 13)$, then $\sum_{r=1}^{\infty} \frac{1}{r \cdot \sqrt{t_r}}$ equals

(A) 1 (B) 2 (C) $\frac{3}{2}$ (D) $\frac{1}{2}$

11. The sum of those integers from 1 to 100 which are not divisible by 3 or 5 is

(A) 2489 (B) 4735 (C) 2317 (D) 2632

12. If a, b, c are in GP, a – b, c – a, b – c are in HP, then the value of a + 4b + c is

(A) 12 (B) 0 (C) 11 (D) 2

More than one choice type

13. The value of $\sum_{r=1}^{n} \frac{1}{\sqrt{a + rx} + \sqrt{a + (r-1)x}}$ is

(A) $\frac{n}{\sqrt{a} + \sqrt{a + nx}}$ (B) $\frac{n}{\sqrt{a} - \sqrt{a + nx}}$ (C) $\frac{\sqrt{a + nx} - \sqrt{a}}{x}$ (D) $\frac{\sqrt{a} + \sqrt{a + nx}}{x}$

14. Let a, x, b be in A.P; a, y, b be in G.P and a, z, b be in H.P. If x = y + 2 and a = 5z, then

(A) $y^2 = xz$ (B) x > y > z (C) a = 9, b = 1 (D) a = 1/4, b = 9/4

15. If 1, $\log_y x$, $\log_z y$, $-15 \log_x z$ are in A.P., then

(A) $z^3 = x$ (B) $x = y^{-1}$ (C) $z^{-3} = y$ (D) $x = y^{-1} = z^3$

PART - II : SUBJECTIVE QUESTIONS

1. In an A.P. of which 'a' is the Ist term, if the sum of the Ist 'p' terms is equal to zero, show that the sum of the next 'q' terms is $-\frac{a(p+q)q}{p-1}$.

2. The number of terms in an A.P. is even ; the sum of the odd terms is 24, sum of the even terms is 30, and the last term exceeds the first by 10½; find the number of terms.

3. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid he dies leaving a third of the debt unpaid. Find the value of the first installment.

4. If the p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively, show that
(q – r) a + (r – p) b + (p – q) c = 0.

5. The sum of first p-terms of an A.P. is q and the sum of first q terms is p, find the sum of first (p + q) terms.

6. If b is the harmonic mean between a and c, then prove that $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$.

7. The value of x + y + z is 15 if a, x, y, z, b are in AP while the value of (1/x) + (1/y) + (1/z) is 5/3 if a, x, y, z, b are in HP. Find a and b.

8. Find the value of $S_n = \sum_{n=1}^{n} \frac{3^n \cdot 5^n}{(5^n - 3^n)(5^{n+1} - 3^{n+1})}$ and hence S_∞.

9. If n is a root of the equation $x^2(1 - ac) - x(a^2 + c^2) - (1 + ac) = 0$ and if n HM's are inserted between a and c, show that the difference between the first and the last mean is equal to ac(a – c).

10. Circles are inscribed in the acute angle α so that every neighbouring circles touch each other. If the radius of the first circle is R, then find the sum of the radii of the first n circles in terms of R and α.

11. Let a, b, c be positive real numbers, then prove that –

(i) $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$

(ii) $\frac{2}{a+b} + \frac{2}{b+c} + \frac{2}{c+a} \geq \frac{9}{a+b+c}$

(iii) $\frac{a^2+b^2}{a+b} + \frac{b^2+c^2}{b+c} + \frac{a^2+c^2}{a+c} \geq a + b + c$

(iv) $\frac{1}{a^3(b+c)} + \frac{1}{b^3(a+c)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}$, if abc = 1

12. Let A, G, H be A.M., G.M. and H.M. of three positive real numbers a, b, c respectively such that $G^2 = AH$, then prove that a, b, c are terms of a GP.

Answers

PART - I

1.	(C)	**2.**	(A)	**3.**	(C)	**4.**	(C)	**5.**	(A)	**6.**	(C)	**7.**	(C)
8.	(A)	**9.**	(B)	**10.**	(A)	**11.**	(D)	**12.**	(B)	**13.**	(AC)	**14.**	(ABC)
15.	(ABCD)												

PART - II

2. 8 terms. Series $1\frac{1}{2}, 3, 4\frac{1}{2}, \ldots$ **3.** Rs. 51 **5.** –(p + q)

7. a = 1, b = 9 OR b = 1, a = 9 **8.** $\frac{3}{4}$ **10.** $\frac{R\left(1 - \sin\frac{\alpha}{2}\right)}{2\sin\frac{\alpha}{2}}\left[\left(\frac{1+\sin\frac{\alpha}{2}}{1-\sin\frac{\alpha}{2}}\right)^n - 1\right]$